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FORESHOCKS AND TIME-DEPENDENT EARTHQUAKE HAZARD ASSESSMENT IN SOUTHERN CALIFORNIA

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Abstract

The probability that an earthquake in southern California ($M \geqq 3.0$) will be followed by an earthquake of larger magnitude within 5 days and 10 km (i.e., will be a foreshock) is 6 ± 0.5 per cent (1 S.D.), and is not significantly dependent on the magnitude of the possible foreshock between $M = 3$ and $M = 5$. The probability that an earthquake will be followed by an $M \geqq 5.0$ main shock, however, increases with magnitude of the foreshock from less than 1 per cent at $M \geqq 3$ to 6.5 ± 2.5 per cent (1 S.D.) at $M \geqq 5$. The main shock will most likely occur in the first hour after the foreshock, and the probability that a main shock will occur decreases with elapsed time from the occurrence of the possible foreshock by approximately the inverse of time. Thus, the occurrence of an earthquake of $M \geqq 3.0$ in southern California increases the earthquake hazard within a small space-time window several orders of magnitude above the normal background level.

Introduction

Previous studies of the nature of foreshocks (e.g., Mogi, 1969; Utsu, 1970; Kagan and Knopoff, 1978; Jones and Molnar, 1979; Bowman and Kisslinger, 1984; Jones, 1984) have concentrated on estimating the rate of foreshock occurrence before moderate and large earthquakes and thus have looked backwards in time (i.e., given a group of main shocks, how many were preceded by foreshocks?) These studies have shown that a significant percentage of earthquakes have been preceded by foreshocks [5 per cent (Bowman and Kisslinger, 1984) to 50 per cent (Mogi, 1969), depending on the tectonic regime and definition of foreshock]. Thirty-five per cent of the moderate earthquakes ($M \geqq 5.0$) in southern California were preceded by foreshocks (Jones, 1984), suggesting that foreshocks could be a useful tool for short-term earthquake hazard assessment. Obviously, whether or not an earthquake was preceded by a foreshock is known by the time the main shock occurs. What is needed for real time earthquake hazard assessment is the probability that a given earthquake will be a followed by a main shock, rather than preceded by a foreshock.

This paper determines the percentage of earthquakes by magnitude that were followed by earthquakes of larger magnitude in southern California in the last 52 yr, and from this the probability that a given earthquake will be a foreshock. The distribution of the temporal spacing between foreshocks and main shocks is also analyzed to provide an estimate of the decay in earthquake hazard with time after a possible foreshock has occurred.

Data

All earthquakes recorded within the area mapped in Figure 1 (from 32°0′N to 36°30′N and 122°0′W to 115°0′W) from 1932 to July 1983 with a local magnitude (M_L) greater than or equal to 3.0 have been considered. The data were obtained from the southern Californian catalog compiled and updated by the California Institute of Technology (Hileman *et al.*, 1973). Only $M \geqq 3.0$ earthquakes have been included because that is the estimated level of completeness of the catalog for this time (Hileman, 1978) and also because most of the foreshocks to damaging earthquakes have been above that magnitude. No earthquake of $M \geqq 5.0$ in southern

California since 1966 when the level of completeness of the catalog is $M = 2.0$ to $M = 2.5$ has had its largest foreshock smaller than 3.0 (Jones, 1984).

Aftershocks were removed from the data set, because the probability that an aftershock will be followed by a larger earthquake is not the objective of this study. This was done by defining an aftershock as an earthquake with a magnitude smaller than that of the main shock within a space-time window whose size is determined by the magnitude of the main shock (e.g., Gardner and Knopoff, 1974). The space-time window used is $(M_{main} - 2.9)*$ 40 days and $(M^3_{main} * 0.2)$ km. Excluding these aftershocks, a total of 4811 earthquakes has been reported in southern California of $M \geqq 3.0$ from 1932 to July 1983.

Each nonaftershock followed by a larger event within 5 days was considered to be a foreshock, so that if multiple foreshocks preceded a main shock, each foreshock

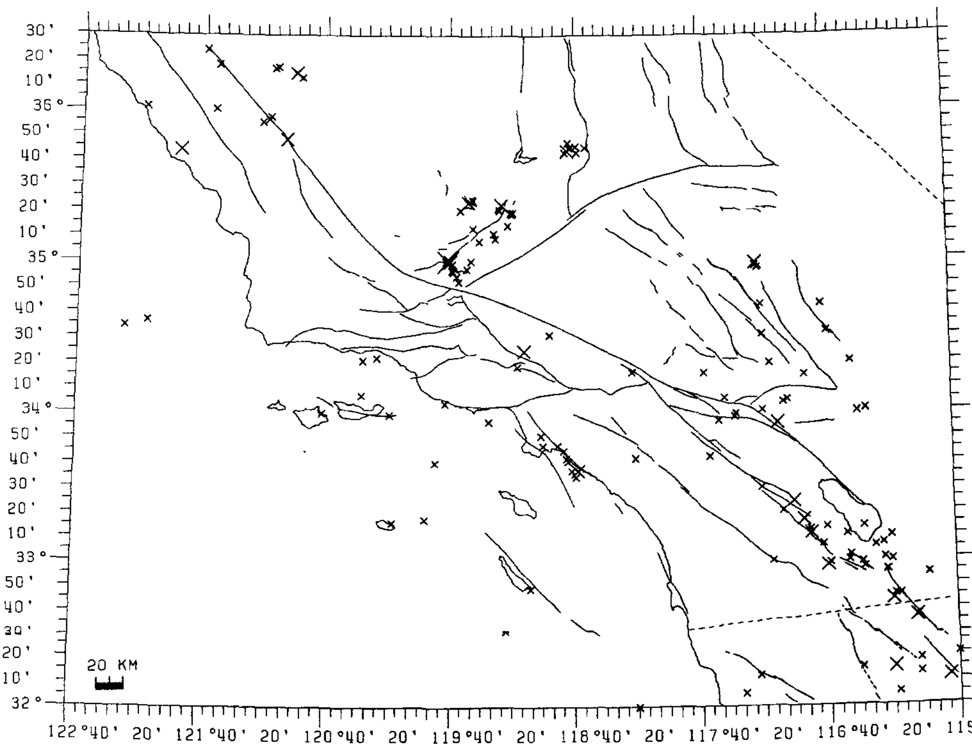

FIG. 1. A map of the area in southern California used for this study. The $M \geqq 5.0$ earthquakes from 1932 to 1983 are also shown.

was counted individually. This does not strongly affect the results, since most main shocks were preceded by only one foreshock as is shown in Table 1 where the number of main shocks preceded by a given number of foreshocks is shown. Moreover, in all 18 sequences with three or more foreshocks of $M \geqq 3.0$, every foreshock has a magnitude equal to or greater than the preceding foreshocks.

ANALYSIS

Identification of foreshocks. A precise definition of foreshocks (that defines a space-time window in which a possible main shock could occur) is required to allow probabilities to be calculated. The optimum space-time window should be large enough that most, but not necessarily all, foreshock-mainshock sequences will be included but small enough that the resulting probabilities of a larger earthquake occurring within that window will be significantly above background level and thus useful for earthquake hazard assessment.

The observed distribution of earthquakes after all nonaftershocks with magnitudes larger than the first earthquake in a 30-km radius circle for 30 days is shown in Figure 2. Clearly, the distribution in space and time of mainshocks after foreshocks is not uniform. Main shocks occur most frequently in the first day after

TABLE 1

THE NUMBER OF MAIN SHOCKS PRECEDED BY A GIVEN NUMBER OF FORESHOCKS AS A FUNCTION OF THE MAGNITUDE OF THE MAIN SHOCK

No. of Foreshocks in Sequence	3.0–3.9	4.0–4.9	5.0–5.9	>6.0	Total
1	102	51	11	2	166
2	14	13	4	0	31
3	3	8	0	0	11
4	0	4	1	1	6
5	1	0	0	0	1

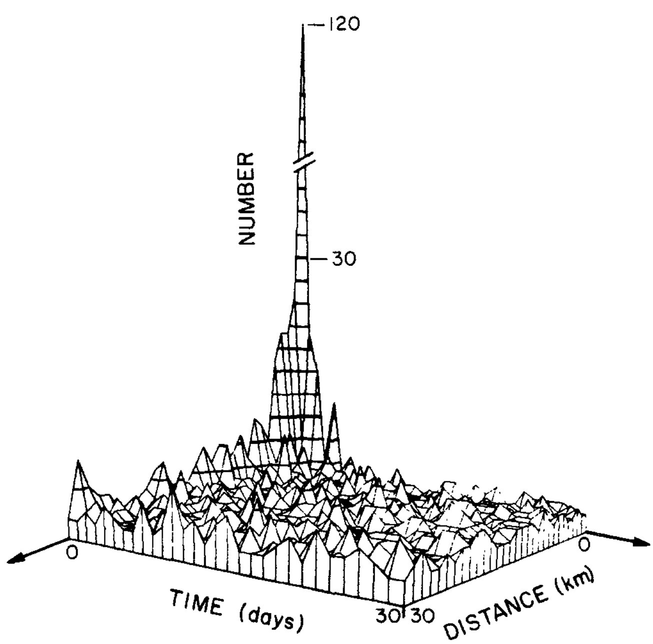

FIG. 2. A three-dimensional histogram of the number of earthquakes larger than the original earthquake recorded within a 30-km radius circle (by km) and 30 days (by day) after all of the 4811 nonaftershocks in the southern California catalog.

the foreshocks and within 1 km of the foreshock epicenter; the rate of occurrence decays quickly with time and distance from the foreshock. Since the purpose here is to maximize the usefulness of a definition of foreshocks rather than to ensure that all foreshock-main shock pairs are included, the space-time window for defining a foreshock was chosen to be 5 days and 10 km. Beyond this window, the occurrence of earthquakes is no more than 100 per cent above the mean rate of occurrence. This spacing between foreshock and main shock is comparable to those found in studies of recent sequences [10 km is slightly larger than previously found and probably results from the larger errors in the locations of earthquakes in the early part of the catalog (Jones, 1984)]. Thus, when the term foreshock is used in this paper, it means that another earthquake, larger than the foreshock, was listed in the catalog within 5 days after the foreshock and with a location less than 10 km from the epicenter of the foreshock.

Probabilities by magnitude. The cumulative number of earthquakes at or above a given magnitude is shown in Figure 3 for: (a) all earthquakes in the data set; (b) all

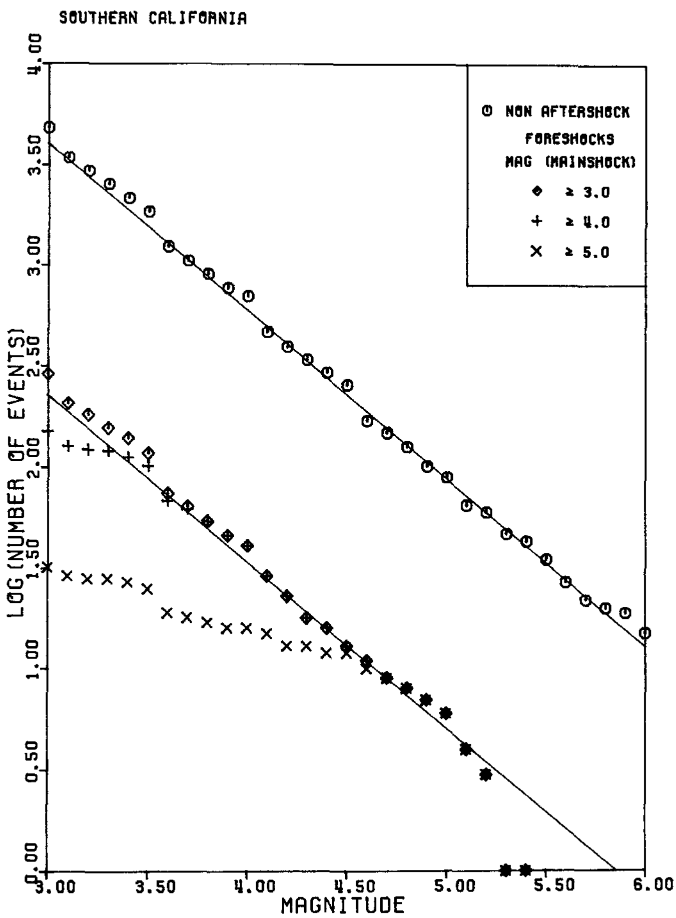

FIG. 3. The cumulative number of nonaftershocks recorded in southern California from 1932 to July 1983 at or above each magnitude level as a function of magnitude. Also shown are the cumulative number of foreshocks, foreshocks to $M \geqq 4.0$ main shocks, and foreshocks to $M \geqq 5.0$ main shocks. Equations (1) and (2) are shown by the solid lines.

foreshocks by the above definition; (c) foreshocks followed by $M \geqq 4.0$ main shocks; and (d) foreshocks followed by $M \geqq 5.0$ main shocks. The obvious offsets in the cumulative number of earthquakes in this figure result from the practice of assigning magnitudes only to the nearest half-unit of magnitude at Caltech until 1943. Using the maximum likelihood method (Aki, 1965), the relationship between the cumulative number of earthquakes, N, and magnitude, M, was found to be,

$$\log(N) = 6.10 - (0.83 \pm 0.02)^*M \tag{1}$$

for all nonaftershocks, and

$$\log(N) = 4.85 - (0.83 \pm 0.10)^*M \tag{2}$$

for the foreshocks. The magnitude coefficients (b values) are the same for both foreshocks and all nonaftershocks (0.83).

Available earthquake catalogs may be used to predict future patterns of seismicity by assuming that the foreshock process is stationary, i.e., that foreshocks are as likely to occur in the next 50 yr as in the last 50 yr. If the occurrence of foreshock-main shock sequences is assumed to be independent of the occurrence of main shocks without foreshocks, the distribution of foreshocks in the set of all earthquakes can be treated as a binomial distribution. The percentage, p, of earthquakes which are foreshocks is then an estimate of the probability that a future earthquake will be a foreshock. The first standard deviation of this estimate is $[p(1 - p)/n]^{1/2}$, where n is the total number of earthquakes (Bevington, 1969).

The probability that an earthquake of $M \geqq 3.0$ in southern California will be followed by an earthquake of a larger magnitude within 5 days and 10 km is approximately 6 ± 0.5 per cent (1 S.D.) (Figure 3). The probability that an earthquake will be a foreshock and the standard deviation of that probability, shown as a function of magnitude in Figure 4, were calculated by dividing the total number of foreshocks at or above a given magnitude by the total number of earthquakes at that or greater magnitude. The standard deviations of the estimates increase with magnitude because of the decrease in size of the data set.

Between $M = 3.0$ and $M = 5.2$, the probability that an earthquake will be a foreshock does not vary much with magnitude. This is consistent with the equivalent b values found for the magnitude distributions of foreshocks and nonaftershocks. If the probability that an earthquake will be a foreshock is determined from equations (1) and (2), the magnitude coefficients in both equations cancel each other, giving a probability of 5.5 per cent, independent of magnitude. Because no foreshock has been reported in southern California larger than $M = 5.4$, the probability that a larger earthquake ($M > 5.4$) will be a foreshock cannot be directly estimated. However, the good fit of both the foreshock and nonaftershock data to straight lines (Figure 2) with similar b values [equations (1) and (2)] suggests that the decline in probability seen in Figure 3 at larger magnitudes results from a lack of data rather than a true change in the behavior of foreshocks.

A dependence on magnitude is seen in the probability that an earthquake will be a foreshock to a potentially damaging $M \geqq 5.0$ main shock (Figure 4). An $M \geqq 3.0$ earthquake has a 1 per cent chance of being followed by an $M \geqq 5.0$ main shock but an $M \geqq 5.0$ earthquake has a 6.5 per cent chance of being followed by another $M \geqq$ 5.0 earthquake. This magnitude dependence is also shown in Figure 5 where the cumulative number of foreshock-main shock pairs with a difference in magnitude greater than or equal to ΔM is plotted against ΔM. A maximum-likelihood fit to this data shows that the number of foreshock-main shock pairs (N) with a magnitude difference greater than or equal to ΔM is

$$\log(N) = 2.38 - (0.75 \pm 0.10)*\Delta M. \tag{3}$$

Thus, the difference in magnitude of foreshocks and main shocks is not 2.0 units of magnitude on the average as has been maintained previously (e.g., Papazachos, 1975). Instead, all magnitude differences are possible with 0.1 units being the most common.

Decay in earthquake hazard with time. The probability that an earthquake will be a foreshock decreases abruptly with elapsed time from the occurrence of the foreshock (Figure 6). The main shock is most likely to occur within the first hour after the occurrence of the foreshock; 26 per cent of the main shocks occur within that time. Within 1 day, 70 per cent of the main shocks will have occurred. A power

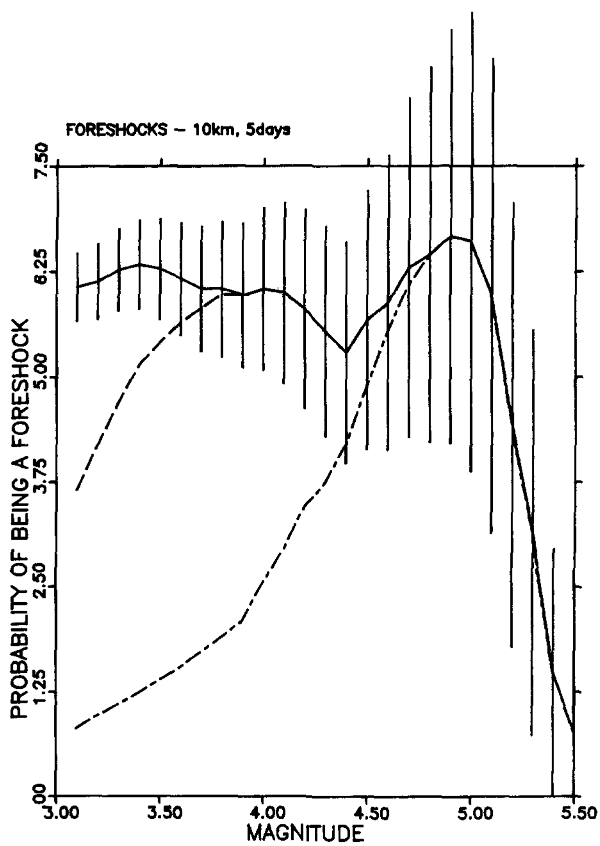

FIG. 4. The probability that an earthquake in southern California will be followed by a larger earthquake within 5 days and 10 km as a function of the magnitude of that earthquake (solid line). The vertical bars show the standard deviation in the estimates of probability for each magnitude level. The dashed line shows the probability of being following by an $M \geqq 4.0$ main shock, and the dotted line shows the probability that an earthquake will be followed by an $M \geqq 5.0$ main shock.

curve fitted to these data gives a rate of decay in hazard of time$^{-0.9}$. The rate of occurrence of main shocks of $M \geqq 4.0$ and $M \geqq 5.0$ also decays with time after the foreshock by time$^{-0.7}$ and time$^{-0.6}$, respectively. This suggestion of a slower time decay for larger main shocks is not significant. The temporal decay in the occurrence of main shocks after foreshocks is thus very similar to the decrease in the occurrence of aftershocks after main shocks which is inversely proportional to time.

Discussion

The occurrence of an earthquake of $M \geqq 3.0$ in southern California increases the probability that a larger earthquake will occur within 10 km and 5 days to 6.0 ± 0.5

per cent. This can be several orders of magnitude above the background level. For instance, approximately 1.5 ($M \geqq 5.0$) earthquakes occur every year in all of southern California which means that the probability that an $M \geqq 5.0$ earthquake will occur anywhere in the region in any given hour is 1.6×10^{-4}. The probability that this $M \geqq 5.0$ event will occur at a given site is much smaller. However, in the 5 days after an $M = 5.0$ earthquake has occurred, there is a 6 per cent chance that another

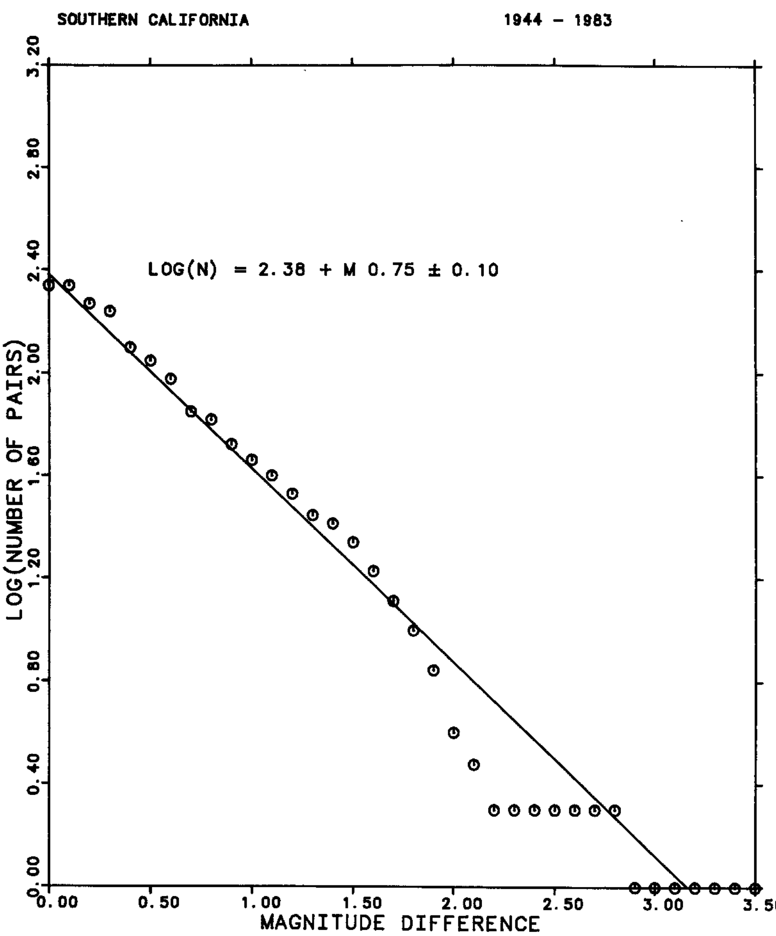

FIG. 5. The cumulative number of foreshock-main shock pairs with a difference in magnitude at or above each level of magnitude difference as a function of difference in magnitude. Only pairs recorded after 1943 (when magnitudes were first given to the nearest 0.1 unit instead of 0.5 unit) are used.

$M \geqq 5.0$ event will occur at the epicenter of the first event. This is 500 times more probable than the background level for the whole region.

More information is available about the probability distribution. The magnitudes of the possible main shocks have a normal b-value distribution above the magnitude of the foreshock so that the probability that an earthquake of magnitude M_m or greater will occur within 5 days after an event of magnitude M_f is

$$P = 0.06 * 10^{-0.75*(M_m - M_f)}. \tag{4}$$

The probability of a main shock occurring decays rapidly with time after the possible foreshock. Twenty-six per cent of the main shocks occur within the first hour, so the probability of a larger earthquake within the first hour after a possible foreshock is $0.06 * 0.26$ or 0.016. After the first hour, the rate of main shock occurrence decays as time$^{-0.9}$. Combining these results, the probability per hour, $P(M_m)$, that an earthquake of $M \geqq M_m$ will occur within 1 hr after time t in hours after an earthquake of $M = M_f$ is

$$P(M_m, t) = 0.016 * 10^{-0.75*(M_m - M_f)} * (t + 1)^{-0.9}. \tag{5}$$

Although this analysis shows that the short-term earthquake hazard increases several orders of magnitude above the background rate after the occurrence of an

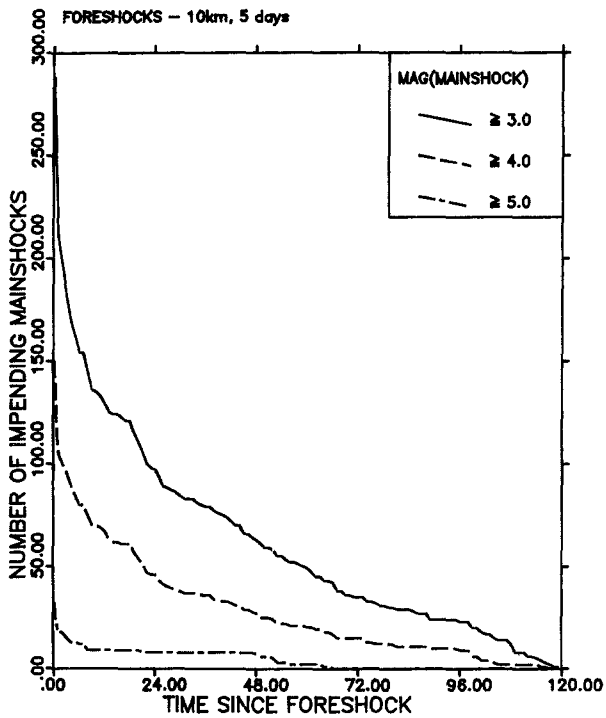

FIG. 6. The number of main shocks still to occur as a function of elapsed time from the foreshock for the 287 foreshock-main shock pairs in the data set.

earthquake, the absolute probability that an earthquake will occur is still quite low. Aki (1981) has shown, however, that increased earthquake hazard resulting from independent precursors may be combined together by considering the probability gain resulting from each precursor. If the probabilities are small, the probability of an earthquake occurring is

$$P = P_o * (P_a/P_o) * (P_b/P_o) * \cdots \tag{6}$$

where P_o is the background rate of occurrence, P_a is the probability due to precursor a, P_b is the probability from precursor b, etc. The ratio of P_a/P_o for foreshocks may be quite high depending on the size of the possible foreshock and the background rate.

The potential usefulness of this method can be shown by considering the probability of a major earthquake occurring on the southern San Andreas fault. Because over a century has passed since the last major earthquake on the southern San Andreas, it is considered one of the most likely sites of a major earthquake within the next few decades (e.g., Sykes and Nishenko, 1984). Half of the $M \geqq 5.0$ strike-slip earthquakes in California between 1966 and 1980 were preceded by foreshocks, so it is quite possible that a major San Andreas earthquake will be preceded by foreshocks. Given the relationship between the magnitudes of foreshocks and main

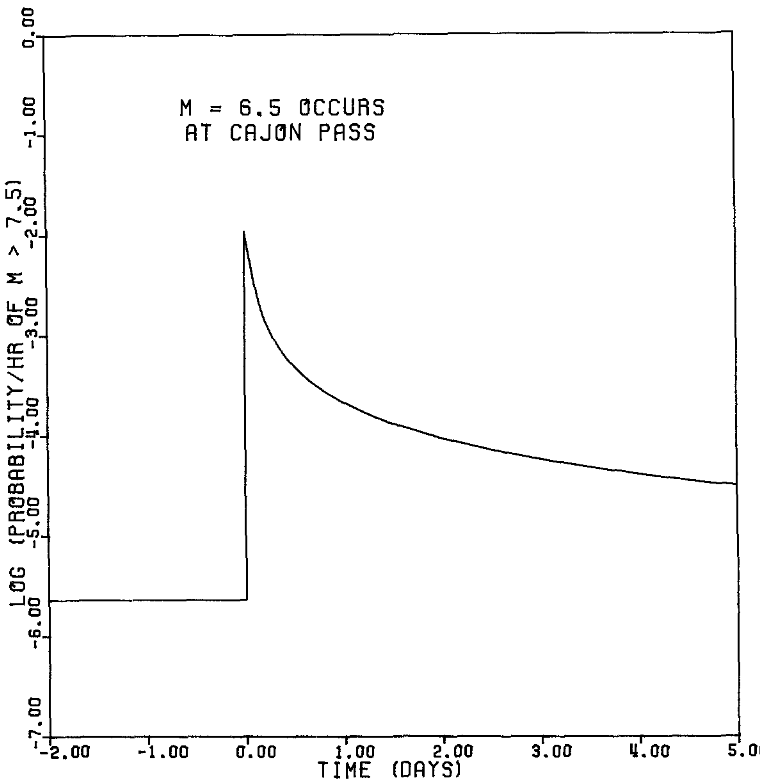

FIG. 7. A schematic plot of the probability per hour of a large ($M \geqq 7.5$) earthquake occurring on the Palmdale section of the southern San Andreas fault as a function of time when an $M = 6.5$ earthquake occurs at Cajon Pass.

shocks shown in equation (3) and Figure 4, it is also likely that this foreshock will be relatively large, perhaps $M \sim 6$ like the foreshocks to the 1857 earthquakes (Sieh, 1978). Thus, it is of great practical importance to estimate the probability that a moderate earthquake on the southern San Andreas fault will be a foreshock to the major plate-rupturing event. Both being at the end of the seismic cycle and the occurrence of the possible foreshock increase the probability of the large earthquake occurring and both factors should be considered in determining the short-term earthquake hazard. This can be done using equation (6) as shown below.

The section of the San Andreas fault from Cajon Pass to Lake Hughes is estimated by Sieh (1984) to have a recurrence interval of 145 yr for $M \geqq 7.5$ earthquakes. This gives a background rate of occurrence (P_o) of 0.007/yr or 7.9×10^{-7}/hr. The last earthquake on this section occurred in 1857, so the present probability of an $M = 7.5$ earthquake (P_r) is estimated at 0.01 to 0.02/yr (Sieh, 1984) or 1.1 to 2.2 $\times$

10^{-6}/hr. If an $M = 5.5$ earthquake were to occur at one end of this section, either at Lake Hughes or at Cajon Pass, the probability of an $M = 7.5$ earthquake occurring in the next hour (P_f) would be 0.0008 [by equation (5)]. [This assumes that the probability of being a foreshock is independent of magnitude as suggested by equations (1) and (2).] Using 2 per cent/yr for the background rate and equation (6), the total probability of an $M = 7.5$ earthquake occurring in the next hour would be

$$P = P_o * (P_r/P_o) * (P_f/P_o) = 0.0019.$$

This procedure also give a probability of 0.7 per cent of the $M = 7.5$ occurring within the next 5 days. The probability of an $M = 7.5$ earthquake occurring after an $M = 6.5$ earthquake in Lake Hughes or Cajon Pass would be, by the same rationale, 1.0 per cent for the first hour and 4.0 per cent for the first 5 days.

The temporal progression of the earthquake hazard on the southern San Andreas fault after the occurrence of an $M = 6.5$ earthquake at Cajon Pass is shown graphically in Figure 7. The background probability per hour of the plate rupturing earthquake is 2.3×10^{-6}/hr. When an $M = 6.5$ earthquake occurs at Cajon Pass, the probability immediately jumps four orders of magnitude to 1.0×10^{-2}/hr. The probability quickly drops as time from the occurrence of the $M = 6.5$ earthquake elapses without the main shock. After 1 day without a main shock, the probability per hour of the large earthquake is down to 2×10^{-4}. After 5 days, the probability is within a factor of ten of the background level.

The limited spatial and temporal extent of this earthquake hazard means that any earthquake preparation measures that were undertaken on this basis would only be required within a limited region and need only be maintained for a few days. However, the transitory nature of the hazard and the high probability of the main shock occurring within the first hour also mean that a decision to undertake any earthquake preparation measures on the basis of these probabilities would have to be made before the possible foreshock occurs. In addition, for this hazard assessment to be of use, information about the location and magnitude of the possible foreshock would have to be processed and disseminated extremely rapidly.

Conclusions

The probability that an $M \geqq 3.0$ earthquake in southern California will be followed by an earthquake of larger magnitude within 5 days and 10 km (i.e., will be a foreshock) has been shown to be 6 ± 0.5 per cent (1 S.D.). This result is not significantly dependent on the magnitude of the possible foreshock at least between $M = 3$ and $M = 5$. The magnitudes of the main shocks fit a b-value distribution above the the magnitudes of the foreshocks so that the most common main shock is 0.1 units larger than the foreshock. The main shock will most likely occur in the first hour after the foreshock, and the probability that a main shock will occur decreases with elapsed time from the occurrence of the possible foreshock by approximately the inverse of time. Thus, the occurrence of an earthquake of $M \geqq 3.0$ in southern California increases the earthquake hazard within a small space-time window several orders of magnitude above the normal background level.

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